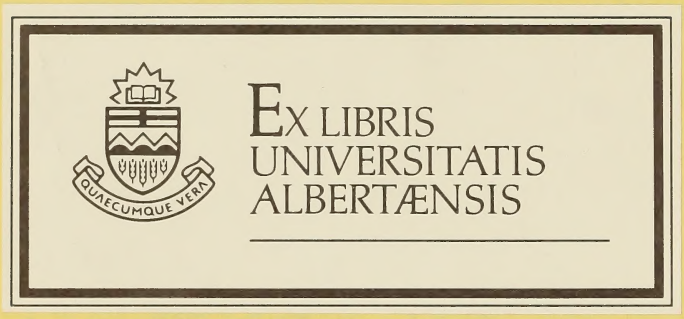
GRADE 12 DIPLOMA EXAMINATION

Mathematics 30

January 1986

Alberta
EDUCATION



GRADE 12 DIPLOMA EXAMINATION
MATHEMATICS 30

DESCRIPTION

Time: 2½ hours

Total possible marks: 65

This is a **CLOSED-BOOK** examination consisting of two parts:

PART A: 52 multiple-choice questions each with a value of 1 mark.

PART B: Five written-response questions for a total of 13 marks.

A mathematics data booklet is provided for your reference. Approved calculators may be used.

GENERAL INSTRUCTIONS

Fill in the information on the answer sheet as directed by the examiner.

For multiple-choice questions, read each carefully and decide which of the choices BEST completes the statement or answers the question. Locate that question number on the answer sheet and fill in the space that corresponds to your choice. USE AN HB PENCIL ONLY.

Example

This examination is for the subject area of

A. Chemistry
B. Biology
C. Physics
D. Mathematics

Answer Sheet

A	B	C	D
①	②	③	●

If you wish to change an answer, please erase your first mark completely.

For written-response questions, read each carefully, show all your calculations, and write your answer in the space provided in the examination booklet.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

DO NOT FOLD EITHER THE ANSWER SHEET OR THE EXAMINATION BOOKLET

The presiding examiner will collect the answer sheet and examination booklet for transmission to Alberta Education.

JANUARY 1986

PART A

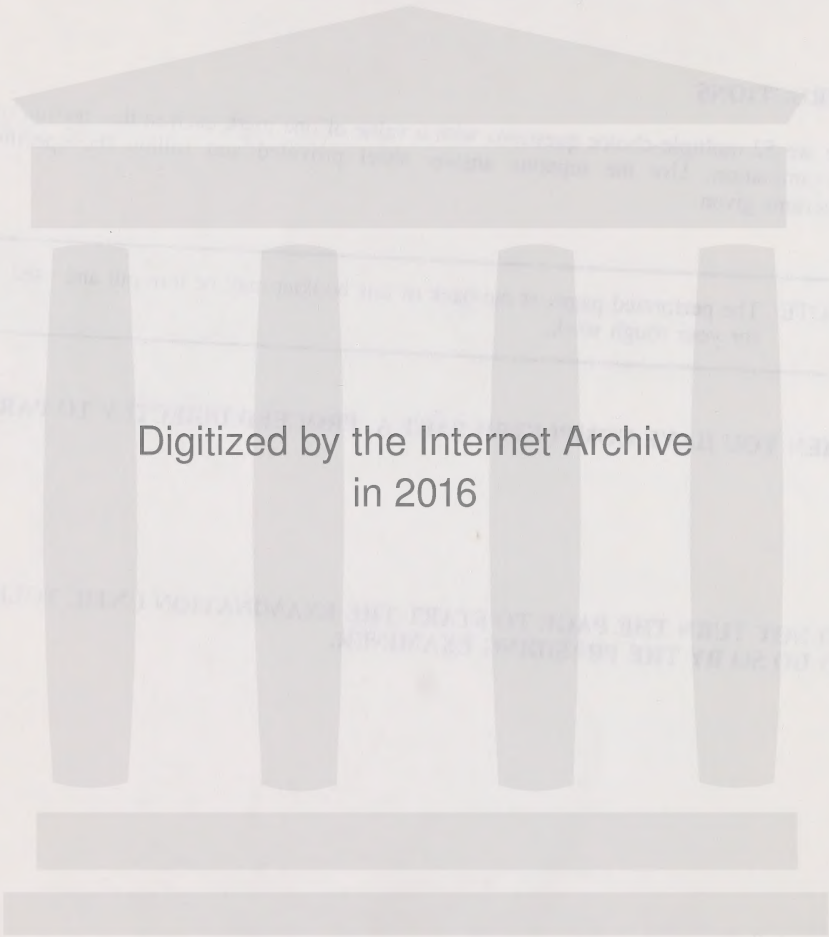
INSTRUCTIONS

There are 52 multiple-choice questions with a value of one mark each in this section of the examination. Use the separate answer sheet provided and follow the specific instructions given.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

WHEN YOU HAVE COMPLETED PART A, PROCEED DIRECTLY TO PART B

DO NOT TURN THE PAGE TO START THE EXAMINATION UNTIL TOLD TO DO SO BY THE PRESIDING EXAMINER.



1. If a person who is 1.6 m tall casts a shadow 1.9 m long, then the angle of elevation of the sun is approximately

   A. 33°
   B. 40°
   C. 50°
   D. 57°

2. A path on a unit circle, with initial point (1, 0) and length $-\frac{7\pi}{3}$, has a terminal point of

   A. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

   B. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

   C. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

   D. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

3. If the terminal point of a path on the unit circle has co-ordinates $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, the value of $\csc\theta$ is

   A. -2

   B. $-\sqrt{3}$

   C. $\frac{2\sqrt{3}}{3}$

   D. $\frac{2\sqrt{3}}{9}$

4. The range of the function $y = 5\sin(2\theta)$ is

A. $\left\{ y \mid -2 \leq y \leq 2 \right\}$

B. $\left\{ y \mid -5 \leq y \leq 5 \right\}$

C. $\left\{ y \mid -\frac{5}{2} \leq y \leq \frac{5}{2} \right\}$

D. $\left\{ y \mid -\frac{2}{5} \leq y \leq \frac{2}{5} \right\}$

5. If $x \sin\theta = 3$ and $x = 4(1 + \sin\theta)$, $x \geqslant 0$, then x is equal to

A. $\frac{\pi}{6}$

B. 0

C. 2

D. 6

6. If $\tan\theta = -\frac{5}{4}$, $\frac{3\pi}{2} < \theta < 2\pi$, then $\sec\theta$ is

A. $-\frac{\sqrt{41}}{4}$

B. $-\frac{\sqrt{41}}{5}$

C. $\frac{\sqrt{41}}{4}$

D. $\frac{\sqrt{41}}{5}$

7. The expression below has been simplified, but leads to a wrong answer.

$$\frac{1}{\tan^2 B} + 1 =$$

Step 1. $\dfrac{\sin^2 B}{\cos^2 B} + 1$

Step 2. $\dfrac{\sin^2 B}{\cos^2 B} + \dfrac{\cos^2 B}{\cos^2 B}$

Step 3. $\dfrac{(\sin^2 B + \cos^2 B)}{\cos^2 B}$

Step 4. $\dfrac{1}{\cos^2 B}$

In which step does the first error occur?

A. 1
B. 2
C. 3
D. 4

8. The expression $\csc(2\pi - \theta)$ is equivalent to

A. $-2\pi \csc \theta$
B. $\csc \theta$
C. $-\csc \theta$
D. $2 \csc \theta$

9. The expression $\cos 80°$ is equivalent to

A. $\cos^2 40° - \sin^2 40°$

B. $\cos^2 40° + \sin^2 40°$

C. $\cos 10°$

D. $\sin 40° \cos 40° + \cos 40° \sin 40°$

10. The graph of $y = \tan\theta$ intersects the graph of $y = -1$ where θ equals

A. $\frac{\pi}{4} + n\pi,\ n \in I$

B. $\frac{\pi}{4} + 2n\pi,\ n \in I$

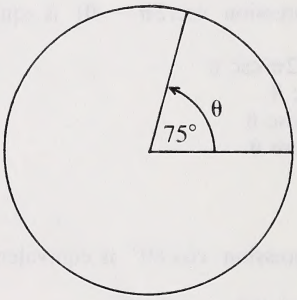
C. $\frac{3\pi}{4} + n\pi,\ n \in I$

D. $\frac{3\pi}{4} + 2n\pi,\ n \in I$

11. If $y = 3\sin(2\theta) + 4$, then the period is

A. 180°
B. 360°
C. 540°
D. 720°

12. In the circle shown at the right, the value of θ in radians is



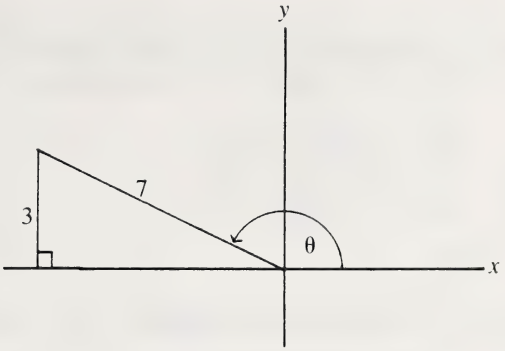
A. $\frac{5\pi}{24}$

B. $\frac{5\pi}{18}$

C. $\frac{5\pi}{12}$

D. $\frac{5\pi}{6}$

13. In the diagram shown at the right, the value of $\cos\theta$ is



A. $-\frac{2\sqrt{10}}{7}$

B. $-\frac{3}{7}$

C. $\frac{3}{7}$

D. $\frac{2\sqrt{10}}{7}$

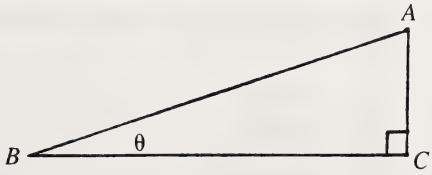
14. In ΔABC, $c = 3$, $a = 4$, and $\angle A = 48°$. To the nearest degree, the measure of $\angle B$ is

A. 34°
B. 82°
C. 98°
D. 116°

15. A greenhouse is 8 m wide. Two rafters of different lengths are placed on opposite walls of equal height and joined above to support the roof. The shorter of the two rafters is 4 m long and is set at an angle of 47° to the horizontal. To the nearest metre, the length of the other rafter is

A. 10 m
B. 9 m
C. 7 m
D. 6 m

16. In ΔABC shown at the right,



BC is 12 cm and $\tan\theta = \frac{1}{3}$.

The area of the triangle is

A. 18 cm^2
B. 24 cm^2
C. 36 cm^2
D. 48 cm^2

17. If the end points of the horizontal diameter of a circle are at $(2, -4)$ and $(10, -4)$, then the end points of the vertical diameter of the same circle are at

A. $(0, 6)$ and $(-8, 6)$
B. $(6, 0)$ and $(6, -8)$
C. $(6, 0)$ and $(6, -4)$
D. $(6, -4)$ and $(6, -8)$

18. The centre of the circle defined by $4x^2 - 24x + 4y^2 + 4y + 5 = 0$ is at

A. $(12, -2)$

B. $\left(3, -\frac{1}{2}\right)$

C. $\left(-3, \frac{1}{2}\right)$

D. $(-12, 2)$

19. The circle that circumscribes the square with vertices at $(2, 1)$, $(6, 1)$, $(6, -3)$, and $(2, -3)$ is defined by

A. $(x + 4)^2 + (y - 1)^2 = 32$

B. $(x - 4)^2 + (y + 1)^2 = 32$

C. $(x + 4)^2 + (y - 1)^2 = 8$

D. $(x - 4)^2 + (y + 1)^2 = 8$

20. The locus of all points in the plane that are equidistant from a fixed line and a fixed point not on the line is

A. a circle
B. an ellipse
C. a parabola
D. a hyperbola

21. For the parabola defined by $x^2 = 9y$, the equation of the directrix is

A. $y = \frac{9}{4}$

B. $x = \frac{9}{4}$

C. $x = -\frac{9}{4}$

D. $y = -\frac{9}{4}$

22. The equation of the directrix of a parabola is $x = -2$. The focus of the parabola is 6 units to the right of the directrix and on the x-axis. The equation of the parabola is

A. $y^2 = 12(x - 1)$

B. $x^2 = 12(y - 1)$

C. $y^2 = 24(x - 4)$

D. $x^2 = 24(y - 4)$

23. The parabola determining the shape of a satellite dish is represented by $x^2 = 6y$, $-1 \leq x \leq 1$. The receiver is placed at the focal point. If all units are in metres, how far from the vertex must the receiver be placed?

A. 0 m
B. 1.5 m
C. 3 m
D. 6 m

24. The length of the major axis of $\frac{x^2}{4} + \frac{y^2}{25} = 1$ is

A. 4
B. 5
C. 10
D. 25

25. Which of the following points lies on the ellipse with foci $(\pm 3, 0)$ and vertices $(\pm 4, 0)$?

A. $\left(1, \frac{\sqrt{42}}{24}\right)$

B. $\left(1, \frac{\sqrt{105}}{4}\right)$

C. $\left(1, \frac{4\sqrt{42}}{7}\right)$

D. $\left(1, \frac{4\sqrt{105}}{105}\right)$

26. An arch 22 m wide at its base forms one half of an ellipse. If the height of the arch at its centre is 10 m, an equation of the ellipse with centre (0, 0) that defines the curve of the arch is

A. $\frac{x^2}{121} + \frac{y^2}{100} = 1$

B. $\frac{x^2}{484} + \frac{y^2}{100} = 1$

C. $\frac{x^2}{121} + \frac{y^2}{25} = 1$

D. $\frac{x^2}{484} + \frac{y^2}{25} = 1$

27. The foci of a hyperbola defined by $\frac{y^2}{144} - \frac{x^2}{25} = 1$ are

A. $(0, \sqrt{119}), (0, -\sqrt{119})$

B. $(0, 13), (0, -13)$

C. $(\sqrt{119}, 0), (-\sqrt{119}, 0)$

D. $(13, 0), (-13, 0)$

28. What are the first three terms of a sequence for which the nth term is $a_n = \dfrac{2n^2 - 1}{n}$?

A. $1, \dfrac{7}{2}, \dfrac{17}{3}$

B. $2, \dfrac{7}{2}, \dfrac{17}{2}$

C. 1, 3, 8

D. 2, 3, 8

29. In an arithmetic series the first term is -12 and the fifteenth term is 44. The sum of the first 15 terms is

A. 240
B. 244
C. 420
D. 448

30. Jim's father decides to give him an allowance of \$1.25 per week for four weeks, and then to raise the allowance by 25¢ per week after each consecutive four-week period. If Jim saves all his allowance every week, how many weeks will it take him to save \$45?

A. 6
B. 15
C. 21
D. 24

31. If the sum of the first n terms of a geometric series is $4^n - 1$, then the nth term is

A. $3(4^n)$

B. $3(4^{n-1})$

C. 4^n

D. $4^n - 4^{n-1} - 1$

32. The sum of the series defined by $\sum_{n=2}^{10} (3n + 2)$ is

A. 160
B. 180
C. 185
D. 200

33. The $\lim_{n \to \infty} \left(\frac{n + 8}{n + 4} \right)$ is

A. 0
B. 1
C. 2
D. infinite

34. An example of an infinite convergent sequence is

A. $10, -5, \frac{5}{2}, -\frac{5}{4}, \cdots, -20\left(-\frac{1}{2}\right)^n, \cdots$

B. $-10, -20, -40, -80, \cdots, -5(2)^n, \cdots$

C. $15 + 10 + 5 + 0 + \cdots + (20 - 5n) + \cdots$

D. $4 - 4 + 4 - 4 + \cdots + (-4(-1)^n) + \cdots$

35. The sum of the infinite geometric series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$ is

A. -2

B. 2

C. $-\frac{2}{3}$

D. $\frac{2}{3}$

36. During the first swing of a pendulum the end point of the pendulum travels 81 cm in one direction. On each successive swing the pendulum travels $\frac{9}{10}$ as far as in the previous swing. The distance the pendulum travels before coming to rest is

A. 1620 cm
B. 1521 cm
C. 810 cm
D. 90 cm

37. The measure of central tendency that is least affected by one extreme value in a set of data is

A. mean
B. range
C. median
D. standard deviation

38. For a standard normal distribution, the area between $z = 1.23$ and $z = -0.75$ is

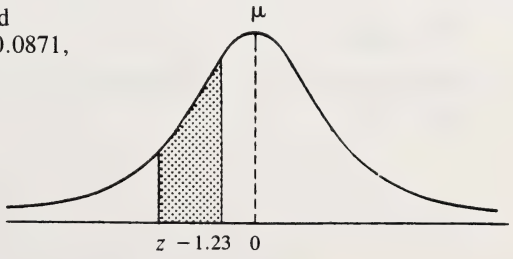
A. 0.1173
B. 0.1844
C. 0.4761
D. 0.6641

39. The masses of 1200 students at a local high school were normally distributed about a mean of 70 kg with a standard deviation of 20 kg. The predicted number of students who weigh more than 90 kg is

A. 190
B. 408
C. 792
D. 1008

40. If the shaded area under the standard normal curve shown at the right is 0.0871, the value of z is

A. -2.01
B. -1.32
C. -1.14
D. -0.41



41. The mean life of a brand of automobile tire is 50 000 km. If the standard deviation is 2000 km, the percentage of tires that would last longer than 48 500 km is

A. 22.66
B. 27.34
C. 72.66
D. 77.34

42. Refer to the table shown at the right. What is the probability that a person's glove size is represented by a prime number?

A. 0.24
B. 0.28
C. 0.49
D. 0.51

Glove Sizes of 240 People	
Size	Frequency
5	10
6	12
7	48
8	74
9	52
10	36
11	8

43. A manufacturer advertises light bulbs that have a mean life of 60 h and a standard deviation of 12 h. Assuming a normal distribution, the probability that a bulb will last between 30 h and 75 h is

A. 0.0994
B. 0.3944
C. 0.4938
D. 0.8882

44. The expression $\left(\frac{2x^2y^{-1}}{y^3} \right)^{-3}$ equals

A. $\frac{x^6}{6y^{12}}$

B. $\frac{x^6}{8y^{12}}$

C. $\frac{y^{12}}{8x^6}$

D. $\frac{y^{12}}{6x^6}$

45. What is the value of $\log_3(27\sqrt{3})$?

A. $9\sqrt{3}$

B. $9\frac{1}{2}$

C. $3\frac{1}{2}$

D. $3\sqrt{3}$

46. If $\log_2(x) = y$ and $\log_4\left(\frac{x}{4}\right) = y$, then the value of x is

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. -4

D. 4

47. An expression equivalent to $-4\log_3(2)$ is

A. $\log_3(-16)$

B. $\log_3(-8)$

C. $\log_3\left(\frac{1}{16}\right)$

D. $\log_3(16)$

48. If the polynomial $x^3 + 2x^2 + kx + 6$ is divisible by $x + 3$, then the value of k is

A. -1
B. -17
C. 2
D. 5

49. If the polynomial $x^3 - 2x^2 - x + 13$ is divided by $x + a$, then the remainder is

A. 11
B. 13
C. $-a^3 - 2a^2 + a + 13$
D. $a^3 - 2a^2 - a + 13$

50. If $P\left(-\frac{5}{8}\right) = 0$, then one factor of the polynomial $P(x)$ must be

A. $5x - 8$
B. $5x + 8$
C. $8x - 5$
D. $8x + 5$

51. If 1 and 2 are x-intercepts of the graph of $y = 2x^3 + ax^2 - 5x + b$, then a and b respectively are

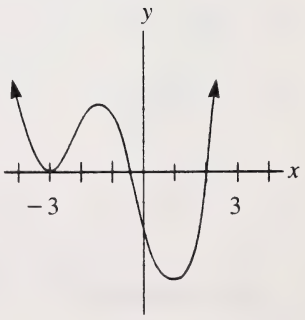
A. $-3, 6$

B. $3, -6$

C. $\frac{5}{3}, -\frac{14}{3}$

D. $-\frac{5}{3}, \frac{14}{3}$

52. The figure shown at the right is a sketch of the graph of



A. $y = (x + 2)(2x - 1)(x - 3)^2$

B. $y = (x + 2)(2x - 1)(x^2 - 3)$

C. $y = (x - 2)(2x + 1)(x^2 + 3)$

D. $y = (x - 2)(2x + 1)(x + 3)^2$

YOU HAVE NOW COMPLETED THE MULTIPLE-CHOICE SECTION OF THE EXAMINATION. PLEASE PROCEED TO THE NEXT PAGE AND ANSWER THE WRITTEN-RESPONSE QUESTIONS IN PART B.

PART B

INSTRUCTIONS

Please write your answers in the examination booklet as neatly as possible.

Show all pertinent calculations and formulas.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

TOTAL MARKS: 13

START PART B IMMEDIATELY

(3 marks) 1. Find the radius of the circle $x^2 + y^2 - 6x + 8y - 11 = 0$.

(3 marks) **2.** A machine that originally cost \$20 000 depreciates 30% during the first year and 10% during each successive year. To the nearest dollar, what is the value of the machine after six years?

(3 marks) 3. The life of a coffee-maker is normally distributed with a mean of 6.2 years and a standard deviation of 2 years. The manufacturer will replace all coffee-makers that last less than 2 years. How many coffee-makers should the manufacturer expect to replace if 3000 are produced?

(2 marks) 4. Solve for x: $\log_5(x + 1) - \log_5(x - 3) = 1$.

(2 marks) 5. Using the synthetic division method, find the quotient and the remainder when $3x^5 - 50x + 4$ is divided by $x - 2$.

YOU HAVE NOW COMPLETED THE EXAMINATION. IF YOU HAVE TIME, YOU MAY WISH TO GO BACK AND CHECK YOUR ANSWERS.

TEAR-OUT SHEET

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

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Jan '86

Table 30

Results for Individual Multiple-Choice Questions
Mathematics 30

Item	Key	Distribution of Responses in %* A	B	C	D	Item	Key	Distribution of Responses in %* A	B	C	D
1	B	5.8	76.0	9.2	8.9	27	B	11.5	62.4	7.5	18.5
2	B	13.6	65.6	11.7	9.1	28	A	79.5	10.7	6.3	3.1
3	C	15.2	7.4	74.0	3.3	29	A	84.8	6.0	6.6	2.6
4	B	11.1	49.0	25.6	14.1	30	D	15.2	11.7	11.0	62.0
5	D	23.0	11.7	23.2	41.4	31	B	5.7	35.7	26.7	31.6
6	C	21.0	15.4	53.9	9.6	32	B	6.3	77.2	7.0	9.3
7	A	78.4	10.8	6.8	3.9	33	B	9.4	56.9	18.7	15.0
8	C	20.3	16.4	52.8	10.2	34	A	59.9	13.6	12.6	13.7
9	A	75.0	10.5	3.6	10.7	35	D	6.3	15.5	8.3	69.8
10	C	15.6	17.7	45.7	20.7	36	C	14.5	18.4	63.7	3.2
11	A	51.9	19.5	16.6	11.7	37	C	13.1	10.3	57.0	19.5
12	C	3.5	3.3	89.0	4.2	38	D	14.5	3.4	12.4	69.6
13	A	63.7	9.4	8.2	18.6	39	A	83.3	11.7	2.5	2.4
14	C	11.8	7.5	79.0	1.6	40	A	46.6	24.5	16.9	11.8
15	D	7.5	11.9	13.4	66.8	41	D	7.4	13.4	14.2	65.0
16	B	9.1	76.0	10.2	4.5	42	B	9.4	49.1	25.7	15.3
17	B	8.5	76.0	8.1	7.1	43	D	4.8	5.6	6.4	83.1
18	B	20.4	62.9	9.7	6.8	44	C	4.0	7.2	82.2	6.5
19	D	7.4	21.8	14.2	56.3	45	C	13.2	5.8	58.8	22.1
20	C	12.7	11.5	65.9	9.9	46	A	52.7	19.0	9.4	18.2
21	D	15.5	10.9	12.4	61.1	47	C	7.6	15.6	68.9	7.8
22	A	57.3	10.6	24.4	7.6	48	A	72.8	10.5	5.9	10.7
23	B	4.8	80.8	10.9	3.3	49	C	9.3	16.4	46.9	27.1
24	C	9.9	18.1	69.0	3.0	50	D	9.2	8.5	15.7	66.4
25	B	10.8	53.2	27.0	8.6	51	A	53.6	26.8	11.6	7.1
26	A	67.7	17.2	12.4	2.7	52	D	10.6	6.5	12.9	68.8

*The sum of the percentages for some questions is less than 100% because the No Response category is not included. This category does not exceed 1.8% for any question.

FOR DEPARTMENT USE ONLY

M1 M2 M3 M4

FOR DEPARTMENT USE ONLY

MATHEMATICS 30

ALBERTA EDUCATION STUDENT ID

NAME: (LAST NAME) (FIRST NAME) DATE OF BIRTH: Y M D SEX:

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SCHOOL CODE: SCHOOL: SIGNATURE: